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Open-chain transfer matrices for AdS/CFT

Rajan Murgan* **and Rafael I. Nepomechie**

Physics Department, P.O. Box 248046, University of Miami
Coral Gables, FL 33124 U.S.A.
E-mail: rmurgan@gustavus.edu, nepomechie@physics.miami.edu

Abstract: We extend Sklyanin's construction of commuting open-chain transfer matrices to the SU(2|2) bulk and boundary S-matrices of AdS/CFT. Using the graded version of the S-matrices leads to a transfer matrix of particularly simple form. We also find an SU(1|1) boundary S-matrix which has one free boundary parameter.

Keywords: Lattice Integrable Models, AdS-CFT Correspondence, Boundary Quantum Field Theory, Exact S-Matrix.

*Current address: Physics Department, Gustavus Adolphus College, Olin Hall, 800 West College Avenue, St. Peter, MN 56082 USA

Contents

1. Introduction

The factorizable SU(2|2)-invariant bulk S-matrix proposed by Beisert [1, 2] plays a central role in understanding integrability in the closed string/spin chain sector of AdS/CFT. Indeed, this S-matrix can be used to derive [1, 3, 4] the all-loop asymptotic Bethe ansatz equations [5] and to compute finite-size effects [6]. (For reviews and further references, see for example ref. [7].)

Integrability also extends to the open string/spin chain sector of AdS/CFT. (See for example [8]–[11] and references therein.) Hofman and Maldacena [12] have proposed boundary S-matrices corresponding to open strings attached to maximal giant gravitons [13] in $AdS_5 \times S^5$. While there has been some subsequent work (see for example [14]–[20]), the study of integrability in the open string/spin chain sector is considerably less-well developed compared with the closed string/spin chain sector. In particular, corresponding all-loop asymptotic Bethe ansatz equations have yet to be derived.

An important prerequisite for deriving such Bethe ansatz equations is to construct a commuting open-chain transfer matrix, which is the main purpose of this note. Sklyanin [21] long ago made the key observation that the transfer matrix should be of the "double-row" form. However, because the bulk S-matrix is not of the difference form and has a peculiar crossing property [22, 2], it is necessary to generalize his construction. Indeed, we argue that the transfer matrix contains an unexpected factor (2.19) which is essential for commutativity. This factor can be removed by working instead with graded versions of the S-matrices.

The SU(2|2) bulk S-matrix has an SU(1|1) submatrix which itself satisfies the Yang-Baxter equation [5, 23]. We find here a corresponding boundary S-matrix which, unlike those found in [12], contains an arbitrary boundary parameter. The simplicity of the SU(1|1) bulk and boundary S-matrices suggests that they can serve as useful toy models of the more complicated SU(2|2) case.

The outline of this paper is as follows. In section 2 we construct two different commuting open-chain transfer matrices. The first, constructed with non-graded S-matrices, contains an unexpected factor; and the second, constructed with graded versions of the S-matrices, does not have this extra factor. In section 3 we present the SU(1|1) boundary S-matrix. We conclude in section 4 with a brief discussion of our results. An appendix contains the SU(2|2) bulk S-matrix and explains some of our notation.

2. Transfer matrix

Bulk and boundary S-matrices are the two main building blocks of the transfer matrix. We assume here that the bulk S-matrix is essentially the one found by Beisert [1] based on SU(2|2) symmetry, but in a basis [2] where the standard Yang-Baxter equation (YBE)

$$S_{12}(p_1,p_2)\,S_{13}(p_1,p_3)\,S_{23}(p_2,p_3) \;= S_{23}(p_2,p_3)\,S_{13}(p_1,p_3)\,S_{12}(p_1,p_2) \tag{2.1}$$

is satisfied. We use the standard convention $S_{12} = S \otimes \mathbb{I}$, $S_{23} = \mathbb{I} \otimes S$, and $S_{13} = \mathcal{P}_{12}\,S_{23}\,\mathcal{P}_{12}$, where $\mathcal{P}_{12} = \mathcal{P} \otimes \mathbb{I}$, $\mathcal{P}$ is the permutation matrix, and $\mathbb{I}$ is the four-dimensional identity matrix. For convenience, this S-matrix is given explicitly in the appendix. For simplicity, we omit the scalar factor. Hence, this matrix has the unitarity property

$$S_{12}(p_1,p_2)\,S_{21}(p_2,p_1) = \mathbb{I}\,, \tag{2.2}$$

where $S_{21} = \mathcal{P}_{12}\,S_{12}\,\mathcal{P}_{12}$, as well as the crossing property [22, 2]

$$C_2(p_2)\,S_{12}(p_1,\bar{p}_2)\,C_2(p_2)^{-1}\,S_{12}(p_1,p_2)^{t_2} = \mathbb{I} f(p_1,p_2)\,, \tag{2.3}$$

where $C(p)$ is the matrix

$$C(p) = \begin{pmatrix} 0 & i\,\mathrm{sign}(p) & 0 & 0 \\ -i\,\mathrm{sign}(p) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \tag{2.4}$$

and the scalar function $f(p_1,p_2)$ is given by

$$f(p_1,p_2) = \frac{\left(\frac{1}{x_1^+} - x_2^-\right)(x_1^+ - x_2^+)}{\left(\frac{1}{x_1^-} - x_2^-\right)(x_1^- - x_2^+)}\,. \tag{2.5}$$

Moreover, $\bar{p} = -p$ denotes the antiparticle momentum, with

$$x^{\pm}(\bar{p}) = \frac{1}{x^{\pm}(p)}\,. \tag{2.6}$$

As we shall see, the peculiar dependence of the charge conjugation matrix $C(p)$ on the sign of p gives rise to a nontrivial factor in the transfer matrix.

We assume here that the right boundary S-matrix $R^-(p)$ is essentially the one found by Hofman and Maldacena [12] for the so-called $Y = 0$ giant graviton brane, but in a basis [16] where the standard (right) boundary Yang-Baxter equation (BYBE) [24, 25]

$$S_{12}(p_1, p_2)\, R_1^-(p_1)\, S_{21}(p_2, -p_1)\, R_2^-(p_2) = R_2^-(p_2)\, S_{12}(p_1, -p_2)\, R_1^-(p_1)\, S_{21}(-p_2, -p_1) \tag{2.7}$$

is satisfied. It is a diagonal matrix given by [16]

$$R^-(p) = \mathrm{diag}\left(e^{-ip}\,, -1\,, 1\,, 1\right)\,. \tag{2.8}$$

As noted in [12],

$$x^{\pm}(-p) = -x^{\mp}(p)\,, \qquad \eta(-p) = \eta(p)\,. \tag{2.9}$$

From the bulk S-matrix, we construct a pair of monodromy matrices

$$\begin{aligned} T_a(p\,;\{p_i\}) &= S_{aN}(p, p_N) \cdots S_{a1}(p, p_1)\,, \\ \widehat{T}_a(p\,;\{p_i\}) &= S_{1a}(p_1, -p) \cdots S_{Na}(p_N, -p)\,, \end{aligned} \tag{2.10}$$

where $\{p_1, \ldots, p_N\}$ are arbitrary "inhomogeneities" associated with each of the N quantum spaces, and the auxiliary space is denoted by a. (As usual, the quantum-space "indices" are suppressed from the monodromy matrices.) These matrices obey the relations

$$\begin{aligned} S_{ab}(p_a, p_b)\, T_a(p_a\,;\{p_i\})\, T_b(p_b\,;\{p_i\}) &= T_b(p_b\,;\{p_i\})\, T_a(p_a\,;\{p_i\})\, S_{ab}(p_a, p_b)\,, \\ S_{ba}(-p_b, -p_a)\, \widehat{T}_a(p_a\,;\{p_i\})\, \widehat{T}_b(p_b\,;\{p_i\}) &= \widehat{T}_b(p_b\,;\{p_i\})\, \widehat{T}_a(p_a\,;\{p_i\})\, S_{ba}(-p_b, -p_a)\,, \\ \widehat{T}_a(p_a\,;\{p_i\})\, S_{ba}(p_b, -p_a) T_b(p_b\,;\{p_i\}) &= T_b(p_b\,;\{p_i\})\, S_{ba}(p_b, -p_a)\, \widehat{T}_a(p_a\,;\{p_i\}) \end{aligned} \tag{2.11}$$

as a consequence of the YBE. The "decorated" right boundary S-matrix given by

$$\mathcal{T}_a^-(p\,;\{p_i\}) = T_a(p\,;\{p_i\})\, R_a^-(p)\, \widehat{T}_a(p\,;\{p_i\}) \tag{2.12}$$

also satisfies the BYBE, i.e.,

$$\begin{aligned} & S_{ab}(p_a, p_b)\, \mathcal{T}_a^-(p_a\,;\{p_i\})\, S_{ba}(p_b, -p_a)\, \mathcal{T}_b^-(p_b\,;\{p_i\}) \\ & \quad = \mathcal{T}_b^-(p_b\,;\{p_i\})\, S_{ab}(p_a, -p_b)\, \mathcal{T}_a^-(p_a\,;\{p_i\})\, S_{ba}(-p_b, -p_a)\,, \end{aligned} \tag{2.13}$$

by virtue of (2.7) and (2.11).

Following Sklyanin [21], we assume that the open-chain transfer matrix is of the double-row form

$$\begin{aligned} t(p\,;\{p_i\}) &= \mathrm{tr}_a\, R_a^+(p)\, \mathcal{T}_a^-(p\,;\{p_i\}) \\ &= \mathrm{tr}_a\, R_a^+(p)\, T_a(p\,;\{p_i\})\, R_a^-(p)\, \widehat{T}_a(p\,;\{p_i\})\,, \end{aligned} \tag{2.14}$$

where the trace is over the auxiliary space, and the left boundary S-matrix $R^+(p)$ is chosen to ensure the essential commutativity property

$$\left[t(p\,;\{p_i\})\,, t(p'\,;\{p_i\})\right] = 0 \tag{2.15}$$

for arbitrary values of p and p'. By repeating the (not short) computation in [21], but now making use of the unitarity and crossing properties (2.2) and (2.3), we find that the commutativity property is indeed obeyed, provided that $R^+(p)$ satisfies the relation

$$\begin{aligned} &S_{21}(p_2,p_1)^{t_{12}}\, R_1^+(p_1)^{t_1}\, C_1(-p_1)\, S_{21}(p_2,\overline{-p_1})^{t_2}\, C_1(-p_1)^{-1}\, R_2^+(p_2)^{t_2} \\ &\quad = R_2^+(p_2)^{t_2}\, C_2(-p_2)\, S_{12}(p_1,\overline{-p_2})^{t_1}\, C_2(-p_2)^{-1}\, R_1^+(p_1)^{t_1}\, S_{12}(-p_1,-p_2)^{t_{12}}\,. \end{aligned} \tag{2.16}$$

In obtaining this result, we also make use of the identity

$$f(p_1,p_2) = f(-p_2,-p_1) \tag{2.17}$$

which is satisfied by the function defined in (2.5). The relation (2.16) can be simplified using again the crossing property (2.3). Eventually, we arrive at

$$\begin{aligned} &S_{12}(p_1,p_2)\, M_1\, R_1^+(-p_1)\, S_{21}(p_2,-p_1)\, M_2\, R_2^+(-p_2) \\ &\quad = M_2\, R_2^+(-p_2)\, S_{12}(p_1,-p_2)\, M_1\, R_1^+(-p_1)\, S_{21}(-p_2,-p_1)\,, \end{aligned} \tag{2.18}$$

where the matrix M is given by

$$M = C(-p)\, C(p)^{-1} = \mathrm{diag}\,(-1\,,-1\,,1\,,1) = M^{-1}\,. \tag{2.19}$$

In obtaining this result, we make use of the identities

$$f(p_1,p_2) = f(\overline{-p_2},\overline{-p_1}) \tag{2.20}$$

and

$$M_1\, S_{12}(p_1,p_2)\, M_2 = M_2\, S_{12}(p_1,p_2)\, M_1\,. \tag{2.21}$$

Comparing the $R^+(p)$ relation (2.18) with the $R^-(p)$ relation (2.7), we conclude that the left boundary S-matrix is given by

$$R^+(p) = M R^-(-p)\,, \tag{2.22}$$

where M is given by (2.19). We emphasize that this matrix M, which arises from the peculiar dependence of the charge conjugation matrix on the sign of the momentum, is essential in order for the transfer matrix (2.14) to have the commutativity property (2.15), which we have verified numerically for small numbers of sites. A formally similar matrix appears in the construction of open-chain transfer matrices for nonsymmetric R-matrices [26].

The matrix M does not appear if we work instead with corresponding graded quantities.[1] Indeed, let us make the parity assignments

$$p(1) = p(2) = 0\,, \qquad p(3) = p(4) = 1\,, \tag{2.23}$$

and define the graded bulk S-matrix by (see, e.g., [3])

$$S^g(p_1,p_2) = \mathcal{P}^g\, \mathcal{P}\, S(p_1,p_2)\,, \tag{2.24}$$

[1]For the generalization of Sklyanin's formalism to graded S-matrices, see for example [27].

where $\mathcal{P}^g$ is the graded permutation matrix

$$\mathcal{P}^g = \sum_{i,j=1}^{4} (-1)^{p(i)p(j)} e_{i\,j} \otimes e_{j\,i} \,, \tag{2.25}$$

and $S(p_1, p_2)$ is given in the appendix. We consider the transfer matrix given by

$$t(p\,;\{p_i\}) = \mathrm{str}_a\, R_a^+(p)\, T_a(p\,;\{p_i\})\, R_a^-(p)\, \widehat{T}_a(p\,;\{p_i\}) \,, \tag{2.26}$$

where str denotes the supertrace, the monodromy matrices are formed as in (2.10) except with the graded S-matrix (2.24) using the graded tensor product (instead of the ordinary tensor product), and $R^-(p)$ is again given by (2.8), which also satisfies the graded BYBE. The transfer matrix (2.26) satisfies the commutativity property (2.15) for $R^+(p)$ given by (2.22) with $M = \mathbb{I}$. That is,

$$t(p\,;\{p_i\}) = \mathrm{str}_a\, R_a^-(-p)\, T_a(p\,;\{p_i\})\, R_a^-(p)\, \widehat{T}_a(p\,;\{p_i\}) \,. \tag{2.27}$$

This transfer matrix evidently has the right structure for formulating the Bethe-Yang equation on an interval with left and right boundaries.[2]

3. SU(1|1) boundary *S*-matrix

The SU(2|2) bulk S-matrix contains an SU(1|1) submatrix which itself satisfies the graded YBE, namely, [5, 23]

$$S(p_1, p_2) = \begin{pmatrix} x_1^+ - x_2^- & 0 & 0 & 0 \\ 0 & x_1^- - x_2^- & (x_1^+ - x_1^-)\frac{\omega_2}{\omega_1} & 0 \\ 0 & (x_2^+ - x_2^-)\frac{\omega_1}{\omega_2} & x_1^+ - x_2^+ & 0 \\ 0 & 0 & 0 & x_1^- - x_2^+ \end{pmatrix} , \tag{3.1}$$

where the parity assignments are $p(1) = 0\,, p(2) = 1$. (We are again not concerned here with overall scalar factors.) Curiously, as already noted by Beisert and Staudacher [5], the YBE holds even without imposing any constraint between $x^+(p)$ and $x^-(p)$, and without specifying $\omega(p)$.

We find that the corresponding right BYBE has the following diagonal solution

$$R^-(p) = \mathrm{diag}\left(a - x^+(p)\,, a + x^-(p)\right) \,, \tag{3.2}$$

where a is an arbitrary boundary parameter. While the appearance of boundary parameters is common for boundary S-matrices associated with affine Lie algebras, we emphasize that no such boundary parameter appears in the SU(2|2) boundary S-matrices [12, 16]. As is the case for the bulk, the BYBE is satisfied without imposing any constraint between $x^+(p)$ and $x^-(p)$ other than (2.9), and without specifying $\omega(p)$ other than

$$\omega(-p) = \omega(p) \,. \tag{3.3}$$

[2] In the closed string/spin chain sector, it is necessary to formulate the Bethe-Yang equation using the graded S-matrix in order to properly implement periodic boundary conditions [3].

The corresponding commuting open-chain transfer matrix is given by (2.26), where the left boundary S-matrix is given by

$$R^{+}(p) = R^{-}(-p)\Big|_{a\mapsto b} = \mathrm{diag}\left(b + x^{-}(p)\,, b - x^{+}(p)\right)\,, \tag{3.4}$$

where b is another arbitrary boundary parameter. The commutativity (2.15) holds for arbitrary $x^{+}(p)\,, x^{-}(p)\,, \omega(p)$ obeying (2.9), (3.3).

4. Discussion

We have found that the SU(2|2) bulk and boundary S-matrices of AdS/CFT can be used to construct a commuting open-chain transfer matrix given by (2.14), where $T_a(p\,;\{p_i\})$ and $\widehat{T}_a(p\,;\{p_i\})$ are given by (2.10), and $R^{+}(p)$ is given by (2.22), which contains the unexpected factor M (2.19). Alternatively, using graded versions of the S-matrices, one can construct the simpler transfer matrix (2.27). Moreover, we have found a new SU(1|1) boundary S-matrix (3.2) which, in contrast to the SU(2|2) case (2.8), contains an arbitrary boundary parameter.

For the SU(2|2) closed chain, a local Hamiltonian can be obtained from the closed-chain transfer matrix

$$t_{\mathrm{closed}}(p\,;\{p_i\}) = \mathrm{tr}_a\, T_a(p\,;\{p_i\}) \tag{4.1}$$

by setting all the inhomogeneities equal $p_i \equiv p_0$, and taking the logarithmic derivative,

$$H_{\mathrm{closed}} = \frac{d}{dp} \ln t_{\mathrm{closed}}(p\,;\{p_i = p_0\})\Big|_{p=p_0}\,. \tag{4.2}$$

As noted by Beisert [1], in contrast to the conventional case, this Hamiltonian depends on the value of p_0, since the bulk S-matrix does not have the difference property. Nevertheless, this Hamiltonian is local, since the S-matrix is regular, $S(p_0, p_0) \propto \mathcal{P}$, and therefore $t_{\mathrm{closed}}(p_0\,;\{p_i = p_0\})$ is the one-site shift operator.

It is not clear whether an analogous local Hamiltonian can be obtained from the open-chain transfer matrix (2.14). Indeed, in contrast to the conventional homogeneous case [21], $t(p_0\,;\{p_i = p_0\})$ is not proportional to the identity. This is due to the fact that

$$\widehat{T}_a(p_0\,;\{p_i = p_0\}) = S_{1a}(p_0, -p_0) \cdots S_{Na}(p_0, -p_0)\,, \tag{4.3}$$

which is not a product of permutation operators, and the fact that $R^{-}(p_0)$ is not proportional to the identity matrix. (This is true even for the conventional inhomogeneous case.) Hence, the naive guess

$$\frac{d}{dp} t(p\,;\{p_i = p_0\})\Big|_{p=p_0} \tag{4.4}$$

does not give a local Hamiltonian; and multiplying (4.4) by $t(p_0\,;\{p_i = p_0\})^{-1}$ does not help.

It would be interesting to determine the eigenvalues and Bethe ansatz equations of the SU(2|2) open-chain transfer matrix. We expect that the SU(1|1) case will serve as a useful warm-up exercise.

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A. The SU(2|2)-invariant bulk S-matrix

We arrange the bulk S-matrix elements into a 16×16 matrix S as follows,

$$S(p_1, p_2) = \sum_{i,i',j,j'=1}^{4} S_{i\,j}^{i'j'}(p_1, p_2)\, e_{i\,i'} \otimes e_{j\,j'}\,, \tag{A.1}$$

where e_{ij} is the usual elementary 4×4 matrix whose (i, j) matrix element is 1, and all others are zero. Although (A.1) is the standard convention, Arutyunov *et al.* use a different convention (see eq. (8.4) in [2]), such that our matrix S is the *transpose* of theirs. The nonzero matrix elements are [2]

$$\begin{aligned}
S_{a\,a}^{a\,a}(p_1, p_2) &= A\,, \qquad & S_{\alpha\,\alpha}^{\alpha\,\alpha}(p_1, p_2) &= D\,, \\
S_{a\,b}^{a\,b}(p_1, p_2) &= \frac{1}{2}(A - B)\,, \qquad & S_{a\,b}^{b\,a}(p_1, p_2) &= \frac{1}{2}(A + B)\,, \\
S_{\alpha\,\beta}^{\alpha\,\beta}(p_1, p_2) &= \frac{1}{2}(D - E)\,, \qquad & S_{\alpha\,\beta}^{\beta\,\alpha}(p_1, p_2) &= \frac{1}{2}(D + E)\,, \\
S_{a\,b}^{\alpha\,\beta}(p_1, p_2) &= -\frac{1}{2}\epsilon_{ab}\epsilon^{\alpha\beta}\, C\,, \qquad & S_{\alpha\,\beta}^{a\,b}(p_1, p_2) &= -\frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta}\, F\,,
\end{aligned}$$
$$S_{a\,\alpha}^{a\,\alpha}(p_1, p_2) = G\,, \quad S_{a\,\alpha}^{\alpha\,a}(p_1, p_2) = H\,, \quad S_{\alpha\,a}^{a\,\alpha}(p_1, p_2) = K\,, \quad S_{\alpha\,a}^{\alpha\,a}(p_1, p_2) = L\,, \tag{A.2}$$

where $a\,, b \in \{1\,, 2\}$ with $a \neq b$; $\alpha\,, \beta \in \{3\,, 4\}$ with $\alpha \neq \beta$; and

$$\begin{aligned}
A &= \frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\,, \\
B &= -\left[\frac{x_2^- - x_1^+}{x_2^+ - x_1^-} + 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^+)(x_1^- x_2^- - x_1^+ x_2^+)}\right]\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\,, \\
C &= \frac{2i x_1^- x_2^- (x_1^+ - x_2^+)\eta_1\eta_2}{x_1^+ x_2^+ (x_1^- - x_2^+)(1 - x_1^- x_2^-)}\,, \qquad D = -1\,, \\
E &= \left[1 - 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^- + x_2^+)}{(x_1^- - x_2^+)(x_1^- x_2^- - x_1^+ x_2^+)}\right]\,, \\
F &= \frac{2i(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^+ - x_2^+)}{(x_1^- - x_2^+)(1 - x_1^- x_2^-)\tilde{\eta}_1\tilde{\eta}_2}\,, \\
G &= \frac{(x_2^- - x_1^-)}{(x_2^+ - x_1^-)}\frac{\eta_1}{\tilde{\eta}_1}\,, \qquad H = \frac{(x_2^+ - x_2^-)}{(x_1^- - x_2^+)}\frac{\eta_1}{\tilde{\eta}_2}\,, \\
K &= \frac{(x_1^+ - x_1^-)}{(x_1^- - x_2^+)}\frac{\eta_2}{\tilde{\eta}_1}\,, \qquad L = \frac{(x_1^+ - x_2^+)}{(x_1^- - x_2^+)}\frac{\eta_2}{\tilde{\eta}_2}\,,
\end{aligned} \tag{A.3}$$

where

$$x_i^\pm = x^\pm(p_i)\,, \quad \eta_1 = \eta(p_1)e^{ip_2/2}\,, \quad \eta_2 = \eta(p_2)\,, \quad \tilde{\eta}_1 = \eta(p_1)\,, \quad \tilde{\eta}_2 = \eta(p_2)e^{ip_1/2}\,, \tag{A.4}$$

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and $\eta(p) = \sqrt{i\,[x^-(p) - x^+(p)]}$. Also,

$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}\,, \quad \frac{x^+}{x^-} = e^{ip}\,. \tag{A.5}$$

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